

Quiz #2

Problems:

1. (30 pt) Let $F : \mathbb{Z} \rightarrow \mathbb{Z}$ be a function defined as $F(x) = 10x$.
 - (a) Prove that F is a group homomorphism.
 - (b) Find $\text{Ker}(F)$, what can you say about F ?
 - (c) Find $\text{Im}(F)$.
2. (10 pt) Let $\phi : G \times X \rightarrow X$ be an action of group G on the set $X = G$ given by conjugation, that $\phi(g, x) = gxg^{-1}$. Suppose x is a fixed point of this action, that means that the conjugacy class of x is equal to $\{x\}$ (a single point). Prove $x \in Z(G)$, i.e. x is in the center.
3. (10 pt) Use the First Isomorphism Theorem to show that $(\mathbb{Z}/30\mathbb{Z}) / \langle [5] \rangle \simeq \mathbb{Z}/5\mathbb{Z}$ (Note: $(\mathbb{Z}/30\mathbb{Z}) / \langle [5] \rangle$ means the group $(\mathbb{Z}/30\mathbb{Z})$ quotiented by its subgroup generated by $[5]$ the class of 5).